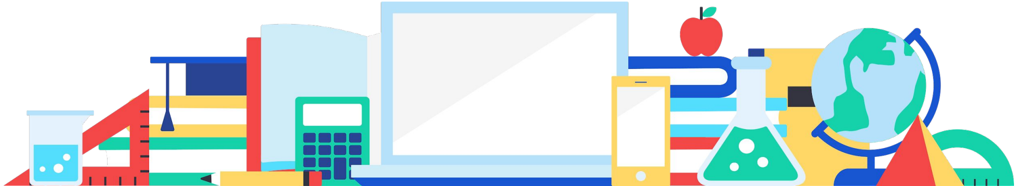
SAT Math Review Guide

Updated February 2024



Topics to Discuss

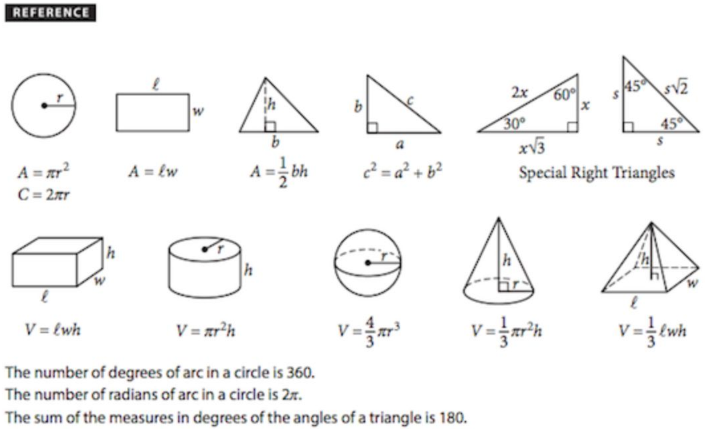
- About the SAT Math Section
- Strategies Before the Test
- Strategies During the Test
- Strategies for Each Question Category
    1. Linear Functions
    2. Quadratic and Exponential Functions
    3. Radicals and Rational Functions
    4. Ratios, Rates, and Proportions
    5. Tables and Graphs
    6. Statistical Analysis
    7. Triangles
    8. Circles

1. About the SAT Math Section

- Logistics
- Types of Content

Spring 2024 SAT Digital Test

- ➔ Starting in Spring 2024, the U.S. SAT will transition to a digital version
- ➔ The digital SAT will be much shorter–2 hours and 14 minutes instead of 3 hours
    - ◆ Math section
        - **44 questions**
        - 2 modules, 35 minutes each
        - 75% four-option multiple-choice, 25% student-produced responses
- ➔ https://satsuite.collegeboard.org/digital/digital-practice-preparation is the official digital practice test
- ➔ Calculators are allowed throughout the Math section
- ➔ There are **no penalties for wrong answers,** so all questions should be answered
- ➔ **Reference information** is provided containing helpful formulas as shown below; however, these should be thoroughly reviewed before taking the exam.
- ➔ The SAT Math Test covers a range of math practices, with an emphasis on **problem solving, modeling, using tools strategically, and using algebraic structure**.



Types of Content

- ➔ There are **4 main content domains** tested on the SAT Math section.
  - ◆ Algebra
  - ◆ Advanced Math
  - ◆ Problem-Solving and Data Analysis
  - ◆ Geometry and Trigonometry
- ➔ These are listed from most tested to least tested, so focus on mastering the top sections before moving on to the others.
- ➔ **Algebra**: analyzing and fluently solving equations and systems of equations; creating expressions, equations, and inequalities to represent relationships between quantities and to solve problems; rearranging and interpreting formulas
- ➔ **Advanced Math**: rewriting expressions using their structure; creating, analyzing, and fluently solving quadratic and higher-order equations; purposefully manipulating polynomials to solve problems
- ➔ **Problem-Solving and Data Analysis**: creating and analyzing relationships using ratios, proportions, percentages, and units; describing relationships shown graphically; summarizing qualitative and quantitative data
- ➔ **Geometry and Trigonometry**: making area and volume calculations in context; investigating lines, angles, triangles, and circles using theorems; and working with trigonometric functions

2. Strategies Before the Test

➔ How to Prepare

How to Prepare

- **Understand and Memorize Formulas**: although the SAT provides reference information, the test does not convey how to use them. When using a formula in practice questions, understand each component of the formula and how to apply it.
    - For example, y = mx + b
        - y = function; vertical position/coordinate on the graph
        - m = slope of the line; $\Delta y/\Delta x$
        - x = horizontal position/coordinate on the graph
        - b = y-intercept; vertical position/coordinate when x is 0
        - This formula is helpful to analyze an equation as a graph or vice versa. It can also be used to determine one of the missing variables.
- **Learn to Perform Basic Calculations**: While you will have a calculator for both modules, being familiar with basic calculations will help you to check your work and increase your confidence in your answers.
    - Completing many practice questions will develop this ability and allot more time for the harder questions, as well as improve speed for the calculator section.
- **Identify a Strategy**: the math problems on both SAT Math sections increase in difficulty as the passage develops.
    - Strategy 1: Take time to ensure all of the easy questions are correct and try to the best of your ability on the rest
    - Strategy 2: Speed up to allot more time for the harder questions at the end
    - Ideally, with practice, the easy questions will be correct and done quickly so that there is ample time for the hard questions as well.

3. Strategies During the Test

- ➔ Generalized Test Taking Strategies for Understanding Math Questions
- ➔ Generalized Test Taking Strategies for Answering Math Questions

Standardized Test Taking Strategies for Understanding SAT Math Questions

- ➔ Read the question all the way through, and **underline the goal**.
  - ◆ Sometimes, questions will ask for the answer to "y", whereas others will ask for "5y + 7". Make sure you know what you are solving for.
- ➔ Go back through the question and change the language from English to Math.
  - ◆ **Assign variables** to the different parts of the problem.
  - ◆ **Draw out diagrams** to model the situation. Fill in these diagrams with the given information.
    - ● Lengths, heights, parallel lines, angles, congruent shapes, etc.
  - ◆ Look for **keywords that indicate mathematical symbols.**

Key Terms	Mathematical Action
Sum, increased by, added to, more than, total of	+
Difference, decreased by, less than, subtracted from	−
Product, times, __ times as much, __ times as many (a number, e.g., "three times as many")	* or x
Divided by, per, __ as many, __ as much (a fraction, e.g., "one-third as much")	/ or ÷
Equals, is, are, equivalent	=
Is less than	<
Is greater than	>
Is less than or equal to	≤
Is greater than or equal to	≥

- ➔ Look at the given information you have written down, and think of how these **variables can be related through a formula** to give the answer.
  - ◆ Units can be very helpful to identify what needs to be added/subtracted or multiplied/divided to give the correct units that correspond to the answer.

Standardized Test Taking Strategies for Answering SAT Math Questions

- **Process of Elimination**: if there are aspects of the answer you are clear on, start by eliminating wrong answers
  - E.g. the value of the slope may be unknown, but it is apparent the correct answer would have a negative slope. In this case, eliminate all answers with a positive slope.
- **Ballpark Answers**: if the answers are far enough apart, making educated guesses may save you time.
  - For example, Instead of attempting to multiply 87 and 123 by hand or immediately resorting to your calculator, you can round 87 to 90 and 123 to 120, which makes the multiplication easier and gives you an idea of what the answer should be approximately.
  - Being able to ballpark answers will also help you cross-check the answers you get from your calculator.
- **Work Backwards from the Answer Choices**: when a question contains variables and is extremely difficult, plugging the answers back into the equation can be easier.
  - Look for the answer choice that makes the question true.
- **Substitute in Numbers for Variables**: similar technique from above, but using your own arbitrary numbers.
  - If the numbers work correctly in both the question and answer choice, that is the correct answer.
  - Make sure to start off by picking easy to calculate numbers, such as 0 or 1. This will ensure easier calculations with lower chances of error, as well as quicker problem solving.

3. Strategies for Each Question Category

- ➔ Linear Functions
- ➔ Quadratic and Exponential Functions
- ➔ Radicals and Rational Functions
- ➔ Ratios, Rates, and Proportions
- ➔ Tables and Graphs
- ➔ Statistical Analysis
- ➔ Triangles
- ➔ Circles

Linear Functions

Additional resources

Solving linear equations and linear inequalities

Interpreting linear functions

Linear equation word problems

Linear inequality word problems

Graphing linear equations

Linear function word problems

- Linear functions refer to functions in which two variables are related to one another in a way that their graph is a straight line.
- **Know the formulas** associated with linear functions and be able to switch between them.
  - $m = \Delta y / \Delta x$
  - $y = mx + b$
  - $(y - y_1) = m(x - x_1)$
- **Understand what each variable represents**, both graphically and in words.
  - m is the slope: on a graph, this determines how steep the line is; in the problem, this is a rate of change and should have units that reflect that (if applicable)
  - b is the y-intercept: on a graph, this is where the line crosses the y-axis; in the problem, this is the value when the x variable is 0 (usually time)
- Comprehend common linear graphs
  - Horizontal lines have a slope of 0
  - Vertical lines have an undefined slope
  - Think about the slope formula: a vertical line has an undefined slope because the denominator is 0
- **Treat inequalities the same way**: do the same process as you would if the sign were an = sign.
  - When **multiplying or dividing a negative number across the inequality** sign, switch its direction.
    - E.g. from $<$ to $>$

Quadratic and Exponential Functions

Additional resources

[Solving quadratic equations]

[Interpreting nonlinear expressions]

[Quadratic and exponential word problems]

[Quadratic and exponential word problems]

- Quadratic functions refer to questions in which a variable to the second power is the highest power term.
- **Taking the square root**: remember to consider both the positive **and** negative answers.
- **Zero product property** states that when two factors are being multiplied and the answer is zero, one of the factors must be zero.
  - E.g. $ab = 0$, so either $a = 0$ or $b = 0$
- **Factor** complex quadratics to visualize solutions more easily.
  - $x^2 + (a + b)x + ab = (x + a)(x + b)$
  - $(a + b)$ is the coefficient of the x term
  - (ab) is the constant term
  - Practice factoring simple quadratics before moving on to harder ones with coefficients in front of the x^2 term
- **Quadratic formula** can take a lot of time, so it should be used sparingly
  - $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$
  - $b^2 - 4ac$ (the part under the square root) is known as the discriminant and can be used to identify the number of distinct, real solutions...
    - If $(b^2 - 4ac) > 0$, there are 2 real solutions
    - If $(b^2 - 4ac) = 0$, there is 1 real solution
    - If $(b^2 - 4ac) < 0$, there are 0 real solutions
- **Exponential functions** all follow the same pattern: $F(t) = a(b)^{ct}$
  - a: initial value
  - b: change
  - c: constant
  - t: time
    - Ensure the units of all components are consistent

Radicals and Rational Functions

Additional resources

Radical and rational exponents

Radical and rational equations

Operations with rational expressions

- Rational functions refer to questions in which exponents can be represented as fractions; radicals are alternative ways to represent this.
- **Review the exponent formulas**: these remain the same even when the exponents are fractions.
  - **Product rule:** $x^m \bullet x^n = x^{m+n}$ **same bases**
  - **Quotient rule:** $\frac{x^m}{x^n} = x^{m-n} \quad x \neq 0$ **same bases**
  - **Negative exponent rule:** $b^{-n} = \frac{1}{b^n} \qquad \frac{1}{b^{-n}} = b^n$
  - **Power rule:** $(x^m)^n = x^{mn}$
  - **Expanded power rule:** $(xy)^m = x^m y^m$ **and** $\left(\frac{ax}{by}\right)^m = \frac{a^m x^m}{b^m y^m} \quad b \neq 0, y \neq 0$
  - **Zero exponent rule:** $x^0 = 1 \quad x \neq 0$
- Understand how exponents can be rewritten as root values.
  - E.g. $x^{1/2}$ is equal to $\sqrt{x}$
    - $\sqrt{x^2} = x$, since the exponent and the square root cancel out
    - $\sqrt{x} = x^{1/2}$
- These questions often times have **extraneous solutions**, which seem like answers, but do not solve the equation if plugged back in
  - In order to check for these, plug answers back into the equation after solving to ensure they give the correct answer.

Ratios, Rates, and Proportions

Additional resources

Ratios, rates, and proportions

Percents

- A **ratio** is a comparison of two quantities, usually one value divided by another.

- A **proportion** is an equality of two ratios.
- Make sure to identify if the ratio or proportion that is being expressed is part-to-part or whole-to-whole
  - E.g. lemons to sugar (part-to-part) or lemons to lemonade (part-to-whole)

- A **percentage** is a ratio out of 100 that represents a part-to-whole relationship.
  - Be able to switch between different types of percentages: ratios, fractions, and decimals
  - Percentages are complementary: if you know 70% of kids are doing one activity, then the remaining 30% of kids are not doing that activity

$$p\% = \frac{p}{100}$$

- A **rate** is the quotient of a ratio where the quantities have different units.
- Rates usually come up in dimensional analysis.
  - Make sure to use rates in the correct direction that cancels out the units you don't want and keeps the units you do.

Tables and Graphs

Additional resources

[Table data]

[Scatterplots]

[Key features of graphs]

- **Two-way frequency tables** include two qualitative variables, one represented by rows and the other represented by columns.
- Table data can be used to find proportions.
  - E.g.. what fraction of students from Mr. Jones class prefer math over english?
    - The answer would be the number of students in Mr. Jones class who prefer math divided by the total number of students in Mr. Jones class
  - If it was asking about all students, you would add the total number students from all classes.
- If there is missing data, fill it in using simple addition and subtraction and the total values.
  - E.g. if there are 40 students who prefer math and 100 students total, then 60 students prefer english.
- When analyzing any type of **graph**, first read the title and both axes.
- Look out for key phrases:

Phrase	Shape of graph
"Increases", "rises", "grows"	Upward trend
"Decreases", "drops", "declines"	Downward trend
"Remains constant", "stops", "stays the same"	Flat trend
"Slowly", "gradually"	Shallow slope
"Rapidly", "quickly"	Steep slope

Statistical Analysis

Additional resources

Data inferences

Center, spread, and shape of distributions

Data collection and conclusions

- A **random sample** drawn from a population is representative of the population.
  - estimate = sample proportion(population)
- When we make reasonable estimates using sample proportions, we can never be 100% certain that the population proportion matches the sample proportion exactly. **Margins of error** let us address the uncertainty inherent to sampling.
  - range = estimate ± margin of error
- **Mean**: average value of a data set
- **Median**: the middle value when data is ordered from least to greatest
  - If there are an even number of values, take the average of the two middle numbers
- **Mode**: the value that appears the most frequently in a data set
- **Range** measures the total spread of the data
  - range = maximum value - minimum value
- **Standard deviation** measures the typical spread from the mean; it is roughly the average distance between the mean and a value in the data set.
  - Larger standard deviations indicate greater spread in the data.
- An **outlier** is a value in a data set that significantly differs from other values.
  - Inclusion of outliers increases the spread of data, leading to larger range and standard deviation.
  - If a very high outlier is removed, the mean of the remaining values will decrease.
  - If a very low outlier is removed, the mean of the remaining values will increase.
  - The mode is not affected.

Triangles

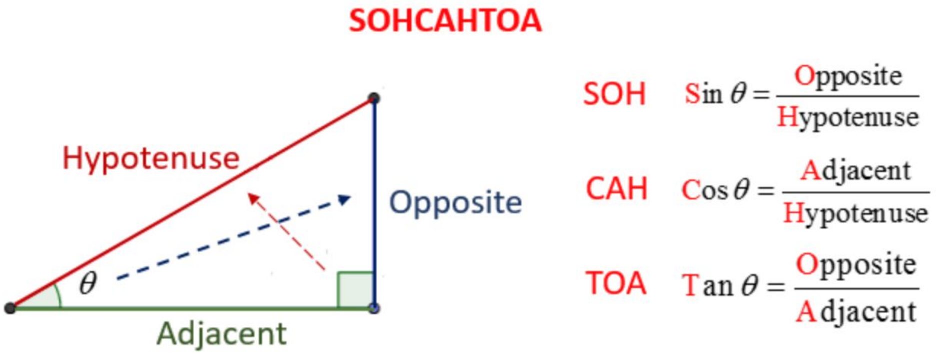
Additional resources

[Right triangle word problems]()

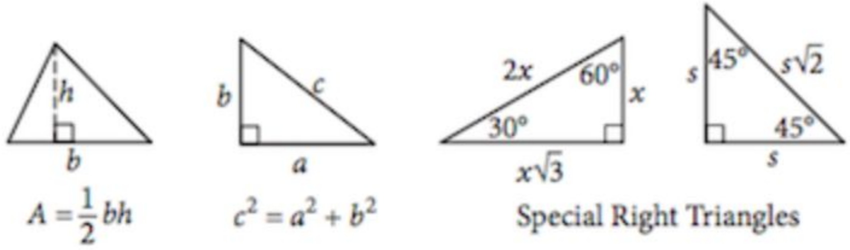
[Congruence and similarity]()

[Right triangle geometry]()

- Trigonometry tests the knowledge surrounding the sides and angles of a triangle
- Understand **SOHCAHTOA**:



- Remember that when two side lengths are the same in any triangle, the angles opposite of those sides will also be the same.
- The SAT Math section provides a few triangle formulas at the beginning of each section. Understand how to use these to find the length of one side of a triangle based off of another given side.



Circles

Additional resources

[Circle theorems](#)

[Circle equations](#)

[Angles, arc lengths, and trig functions](#)

- ➔ The SAT Math section provides a few circle formulas at the beginning of each section. Understand how to use these to find the length or area of a segment of the circle
- ➔ For both length and area, **proportions** can be used. Set the number of degrees of the whole circle (360) over the whole area or circumference, which can be found using the given formulas.
    - ◆ If using area, sector area can be found
    - ◆ If using circumference, sector length can be found
- ➔ Set it equal to the sector's degrees, and solve for the unknown.

$$A = \pi r^2$$
$$C = 2\pi r$$

- ➔ The sum of central angle measures in a circle is **360°**.
- ➔ Since all radii have the same length, any triangle that contains two radii is an **isosceles triangle**.